



PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two:
Calculator-assumed

Your Name: _____

Your Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Mark	Max
8		8	15		10
9		11	16		15
10		8	17		7
11		12	18		7
12		8			
13		10			
14		4			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	11	11	100	100	67
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 11 Information Handbook 2021*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed**(100 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 {1.1.1, 1.1.2, 1.1.4}**(8 marks)**

A family of two adults and four children purchase 6 seats in a row for an Eagles football match. The seats are randomly allocated to members of the family.

(a) How many arranged ways can they be seated? (1 mark)

(b) How many ways can they be seated if they must sit in the order of their ages? (1 mark)

(c) How many ways can they be seated if a child must be sitting on either end? (2 marks)

(d) How many ways can they be seated if the two adults must sit together? (2 marks)

(e) How many ways can they be seated if the two youngest children do not sit together? (2 marks)

Question 9 {1.1.5, 1.1.7}

(11 marks)

Perth Modern School is sending a mixed volleyball team of 6 players to a State-wide competition. If there are 7 girls and 5 boys to choose from.

(a) How many different ways could the team be selected? (1 mark)

(b) How many ways can the team containing 4 girls and 2 boys be selected? (2 marks)

(c) One of the girls, Mekides, is captain and must be chosen. How many teams of 6, containing Mekides, with a total of 4 girls and 2 boys can be selected? (2 marks)

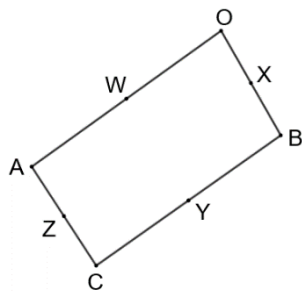
(d) How many teams of 6, containing at least 2 boys and at least 2 girls, can be selected? (3 marks)

(e) A photograph is to be taken of a team of 3 boys and 3 girls selected from the total and then seated in a row for a photograph. How many possible photographs could be taken? (3 marks)

Question 10 {1.2.12, 1.3.17}

(8 marks)

Let $OACB$ be a parallelogram and let W, X, Y and Z be midpoints of the sides as indicated in the diagram below.



Let $\mathbf{a} = \overrightarrow{OA}$ and $\mathbf{b} = \overrightarrow{OB}$.

(a) Write expressions for \overrightarrow{WX} and \overrightarrow{XY} in terms of \mathbf{a} and \mathbf{b} .

(3 marks)

(b) Use a vector method to prove that if $WXYZ$ is a rhombus, then $OACB$ is a rectangle.

(5 marks)

Question 11 {1.2.1, 1.2.2, 1.2.6, 1.2.7}**(12 marks)**

Relative to a fixed origin O , the unit vectors \mathbf{i} and \mathbf{j} are pointing due east and due north, respectively. The velocity of the particle, \mathbf{v} in m/s, at time t in seconds after a given instant is

$$\mathbf{v} = (2 - 3t)\mathbf{i} + (3t - 6)\mathbf{j}.$$

(a) Find the speed of the particle when $t = 0$. (3 marks)

(b) Determine the bearing on which the particle is moving when $t = 4$. (3 marks)

(c) Calculate the value of t when the particle is moving:

i. parallel to \mathbf{i} (2 marks)

ii. parallel to $7\mathbf{i} - 5\mathbf{j}$. (4 marks)

Question 12 {1.2.4}

(8 marks)

Two forces F_1 and F_2 are acting on a particle so that the resultant of the two forces has a magnitude of 120 N acting on a bearing of 140° . F_1 acts due North and has a magnitude of 90 N.

(a) Represent F_2 as a vector in component form, to two decimal places. (4 marks)

(b) Calculate the magnitude of F_2 and the direction of this force, giving the answer as a three-digit bearing. (4 marks)

Question 13 {1.2.10, 1.2.11, 1.2.13}

(10 marks)

Relative to a fixed origin O , the points A and B have respective position vectors $3\mathbf{i} - \mathbf{j}$ and $2\mathbf{i} - 4\mathbf{j}$.

(a) Show that \overrightarrow{OA} and \overrightarrow{AB} are perpendicular. (3 marks)

(b) A line is drawn through the points A and B . Point C lies on this line so that the area of the triangles OAB and OBC are equal. Determine the position vector of C . (2 marks)

(c) Use a vector method, find the shortest distance from B to the line drawn from the origin through point C . (5 marks)

Question 14 {1.1.6}

(4 marks)

A bag contains four blue, six red and seven green balls. How many balls must be chosen at random to guarantee that you will obtain:

(a) at least two balls of the same colour?

(1 mark)

(b) at least three balls of the same colour?

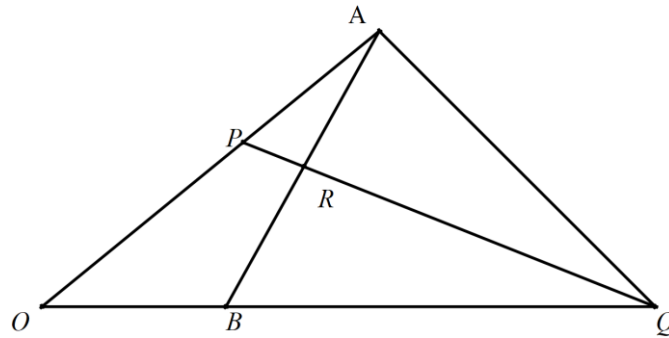
(1 mark)

(c) at least six balls of the same colour?

(2 marks)

Question 15 {1.2.2 – 1.2.12}**(10 marks)**

Consider the figure below showing triangle OAQ which is not drawn to scale. The point P lies on OA such that $OP:OA = 3:5$ and the point B lies on OQ such that $OB:OQ = 1:3$



Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$

(a) Given $\overrightarrow{AR} = h\overrightarrow{AB}$ where h is a scalar, show that $\overrightarrow{OR} = (1 - h)\mathbf{a} + \frac{1}{3}h\mathbf{b}$ (3 marks)

(b) Given $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a scalar, show that $\overrightarrow{OR} = \frac{3}{5}(1 - k)\mathbf{a} + k\mathbf{b}$ (3 marks)

Question 15 continued

(c) Determine the value of k and the value of h

(3 marks)

(d) Determine the ratio of $PR:PQ$

(1 mark)

Question 16 {1.2.1 – 1.2.5}

(15 marks)

(a) Points A, B and C have position vectors $\overrightarrow{OA} = 5\mathbf{i} - 7\mathbf{j}$, $\overrightarrow{OB} = 8\mathbf{i} - 15\mathbf{j}$ and $\overrightarrow{OC} = -4\mathbf{i} + k\mathbf{j}$. Find the **exact** value(s) of k if

i. A, B and C are collinear. (3 marks)

ii. the distance from A to B is half the distance from B to C . (3 marks)

(b) Consider the 2-dimensional vectors $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} + 6\mathbf{j}$ and $\mathbf{c} = x\mathbf{i} + y\mathbf{j}$. Explain why there can be no solutions for x and y satisfying both $\mathbf{a} \cdot \mathbf{c} = 0$ and $\mathbf{b} \cdot \mathbf{c} = 0$.

(3 marks)

Question 16 continued

(c) XYZ is an equilateral triangle. If $\overrightarrow{XY} \cdot \overrightarrow{XZ} = 8$, determine

i. $\overrightarrow{ZY} \cdot \overrightarrow{ZX}$

(2 marks)

ii. $\overrightarrow{XY} \cdot \overrightarrow{YZ}$

(2 marks)

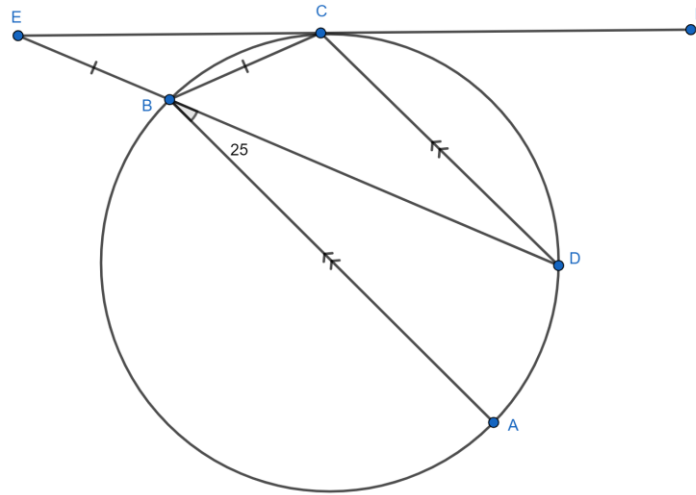
iii. $|\overrightarrow{XY}|$

(2 marks)

Question 17 {1.3.11 – 1.3.15}

(7 marks)

Consider the diagram below (not to scale) showing a circle with diameter AB . Points C and D also lie on the circle and the line EF is a tangent to the circle at point C . AB is parallel to CD and $BC = BE$. EBD is a secant to the circle and angle ABD is 25°



(a) Find the size of $\angle BEC$

(3 marks)

(b) Find the size of $\angle ACF$

(3 marks)

(c) Using relevant circle theorems, find the exact length of the base of $\triangle BCE$ given that $EB = 2 \text{ cm}$ and $EB:BD = 1:2.5$

(1 mark)

Question 18 {1.2.14}**(7 marks)**

At 10 am, electronic Hot Wheels A and B are launched from their locations at $(7i + 3j) m$ and $(-i - 0.8j)m$ with constant velocities $(-i + 0.25j) m/s$ and $(i + 1.2j)m/s$ respectively.

- (a) Write vector expressions for $r_A(t)$, the position vector of A at t second, and $r_B(t)$, the position vector of B at t second. (2 marks)

- (b) Write a vector expression for ${}^A r_B(t)$, the relative position of A with respect to B at t second. (1 mark)

- (c) Hence, show that the cars will collide if these velocities are maintained. State when and where the collision occurs. (4 marks)

Additional working space

Question number: _____

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